

Discrete Mathematics

Recitation Course 9

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9-1

Graphs and Graph Models

9-1 Ex.12

- Let G be an undirected graph with a loop at every vertex. Show that the relation R on the set of vertices of G such that uRv iff there is an edge associated to $\{u, v\}$ is a symmetric, reflexive relation on G .
- Since the graph is undirected, uRv means vRu , thus the relation is symmetric
- The relation is reflexive because the loops at every vertex

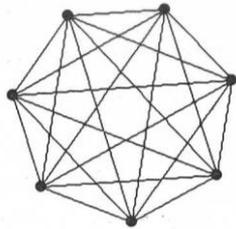
9-2

Graph Terminology
and Special Types of Graphs

9-2 Ex.20

- Draw these graphs

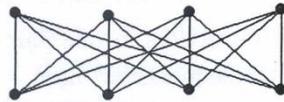
- a) K_7



- b) $K_{1,8}$



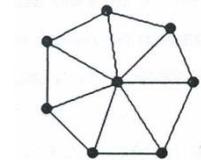
- c) $K_{4,4}$



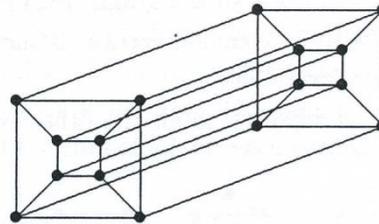
- d) C_7



- e) W_7



- f) Q_4



9-2 Ex.26

- For which values of n are these graphs bipartite?
 - a) K_n
 - b) C_n
 - c) W_n
 - d) Q_n
-
- K_2 only
 - $n \geq 3$ and n is even
 - none
 - for all $n \geq 1$

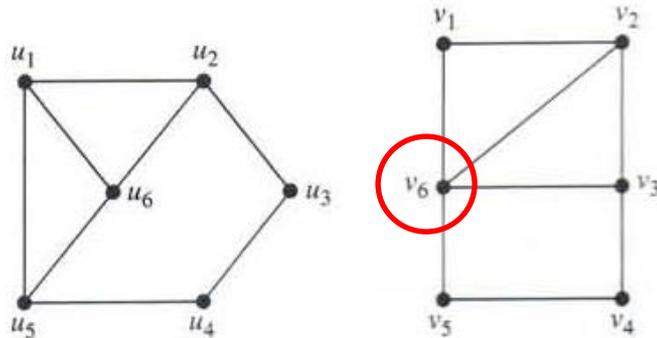
9-3

Representing Graphs
and Graph Isomorphism

9-3 Ex.34-44

- Determine whether the given pair of graphs is isomorphic

- 40)



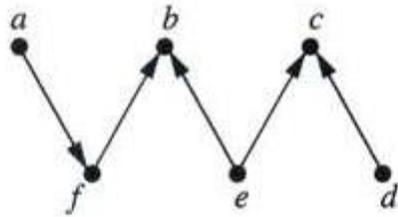
9-4

Connectivity

9-4 Ex.12

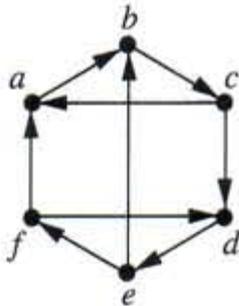
- Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected

• a)



no path from f to a ,
weakly connected

• b)

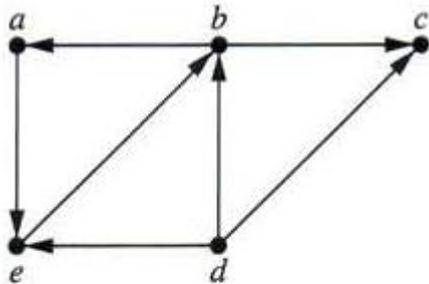


strongly connected

9-4 Ex.14

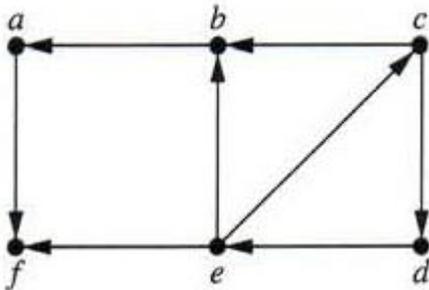
- Find the strongly connected components of each of these graphs

a)



$\{a, b, e\}, \{c\}, \{d\}$

b)



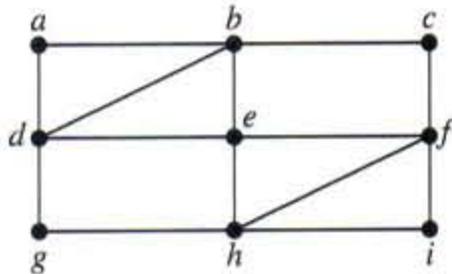
$\{a\}, \{b\}, \{c, d, e\}, \{f\}$

9-5

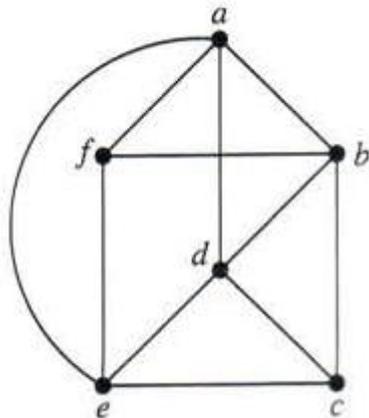
Euler and Hamilton Paths

9-5 Ex.2, 4

- Determine whether the given graph has an Euler circuit or path



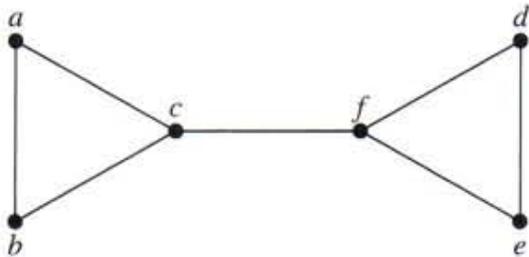
$a, b, c, f, i, h, g, d, e, h, f, e, b, d, a$



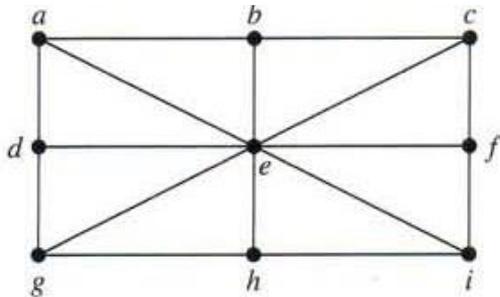
$f, a, b, c, d, e, f, b, d, a, e, c$

9-5 Ex.30, 36

- Determine whether the given graph has an Hamilton circuit



none



$a, d, g, h, i, f, c, e, b, a$

9-6

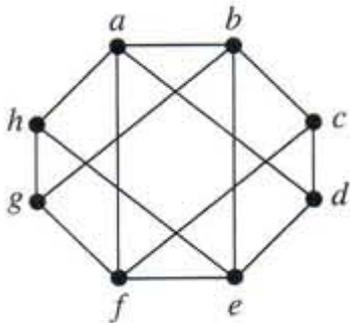
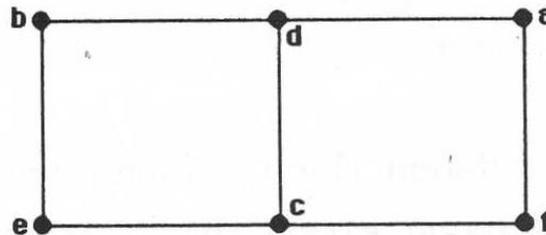
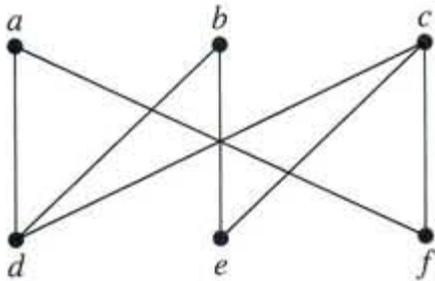
Shortest-Path Problems

9-7

Planar Graphs

9-7 Ex.6, 8

- Determine whether the given graph is planar.
- If so, draw it so that no edges cross.



Theorem 2:

A graph is nonplanar iff it contains a subgraph homeomorphic to $k_{3,3}$ or k_5
 $k_{3,3}: \{a, c, e\}, \{b, d, f\}$

9-8

Graph Coloring

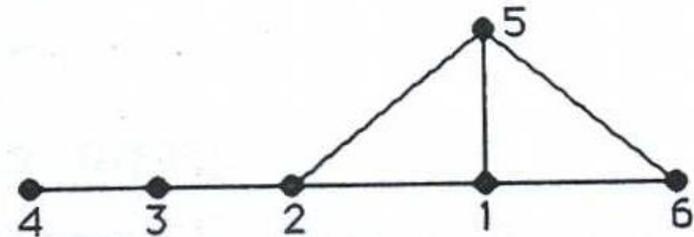
9-8 Ex.13

- Which graphs have a chromatic number of 1?
- Graphs with no edges

9-8 Ex.18

- How many different channels are needed for six stations located at the distances shown in the table, if two stations cannot use the same channel when they are within 150 miles of each other?

	1	2	3	4	5	6
1	—	85	175	200	50	100
2	85	—	125	175	100	160
3	175	125	—	100	200	250
4	200	175	100	—	210	220
5	50	100	200	210	—	100
6	100	160	250	220	100	—



3 colors are necessary
and sufficient !